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FLOW PATTERN, VOID FRACTION AND PRESSURE DROP OF REFRIGERANT TWO-PHASE FLOW IN A HORIZONTAL PIPE—II: ANALYSIS OF FRICTIONAL PRESSURE DROP

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Abstract—Two-phase flow in horizontal pipe was analyzed with simplified models for annular and stratified flow. The velocity profiles for the liquid and gas phase were described with the Prandtl mixing length. From this analysis, the frictional pressure drop was calculated with the modified Baker map for flow pattern transition. The intermediate region, i.e. wavy flow, was interpolated between annular and stratified flow. Comparison of this analysis with existing experimental data of refrigerants showed good agreement.

1. INTRODUCTION

In the part I of this article by Hashizume (1983a), described was experimental equipment for obtaining data on the flow pattern, void fraction and pressure drop of refrigerant two-phase flow in a horizontal pipe, and experimental data were presented. Based on these data, a flow pattern map, a modified Baker map, was proposed by Hashizume (1983b). In this paper, models for annular and stratified two-phase flow in a horizontal pipe are presented and analyzed. From these models, the frictional pressure drop can be calculated for each flow pattern region. The transition region between them is covered by linear interpolation on a log–log plot between the two extremes according to the proposal by Bandel (1973) and Bandel & Schlünder (1973, 1974). The flow pattern transitions are determined by the modified Baker map, whereas Bandel determined these transitions empirically from the comparison of their pressure drop calculation method with experimental data. The calculation method for frictional pressure drop based on this analysis with flow pattern transition according to the modified Baker map is compared with refrigerant data presented in part I and also by Chawla (1967) and Bandel (1973).

2. MODELING FOR HORIZONTAL TWO-PHASE FLOW

2.1 *Falling film flow*

Typical flow patterns in horizontal two-phase flow are annular and stratified flow. They consist of a continuous liquid (film) flow and a gas (core) flow. As the first step of two-phase flow analysis, a falling film flow is considered, which would have similar characteristics to liquid film flow in two-phase flow. Since they both have a free surface as a boundary, their characteristics are supposed to be different from those of single-phase flow.

Figure 1 shows the model for analysis. Along a vertical flat plate flows a turbulent liquid film due to gravity, where the velocity profile is considered to consist of a laminar sublayer and a turbulent layer. The liquid surface is assumed to be smooth.

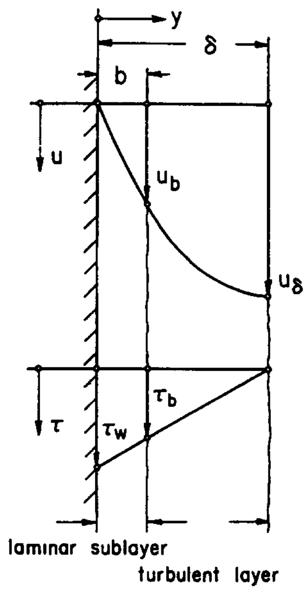


Figure 1. Model for falling film.

The velocity profile in the film and the mass flow rate can be expressed in dimensionless forms as follows:

$$u^+ = y^+, \text{ for } 0 \leq y^+ \leq b^+, \quad [1]$$

$$u^+ = u_b^+ + \frac{1}{m}\left(2\sqrt{1 - y^+/\delta^+} - 2\sqrt{1 - b^+/\delta^+} + \ln\frac{1 - \sqrt{1 - y^+/\delta^+}}{1 + \sqrt{1 - y^+/\delta^+}} - \ln\frac{1 - \sqrt{1 - b^+/\delta^+}}{1 + \sqrt{1 - b^+/\delta^+}}\right), \text{ for } b^+ \leq y^+ \leq \delta^+, \quad [2]$$

$$\mathrm{Re}_L = \int_0^{\delta^+} u^+ \, dy^+, \quad [3]$$

where $u^+(=u/u^*)$ represents the dimensionless velocity, $u^*(=\sqrt{g\delta})$ the friction velocity, g the acceleration due to gravity, y^+ $(=\rho_L u^* y/\mu_L)$ the dimensionless distance, ρ_L the liquid density, μ_L the liquid viscosity, b^+ $(=\rho_L u^* b/\mu_L)$ the dimensionless thickness of the laminar sublayer, δ^+ $(=\rho_L u^* \delta/\mu_L)$ the dimensionless liquid film thickness, Re_L $(=G/\mu_L)$ the liquid Reynolds number and G the mass flow rate per unit width.

The velocity profile for the turbulent layer is described using the Prandtl mixing length,

$$\tau = \rho_L l^2 \left(\frac{\mathrm{d}u}{\mathrm{d}y}\right)^2, \quad [4]$$

$$l = my, \quad [5]$$

where τ represents the shear stress and m the mixing length constant. From [1–3], the dimensionless film thickness δ^+ can be calculated for given Reynolds number Re_L, when the value for m is known. For dimensionless thickness of the laminar sublayer, according to Ueda (1967) and Ueda & Hanaoka (1967), $b^+ = 7$ is assumed to be valid for falling film flow.

Figure 2 shows the results. We can see here that the calculated δ^+ agrees with the empirical correlation after Brauer (1956), if the value of 0.18 is taken for m. The value of

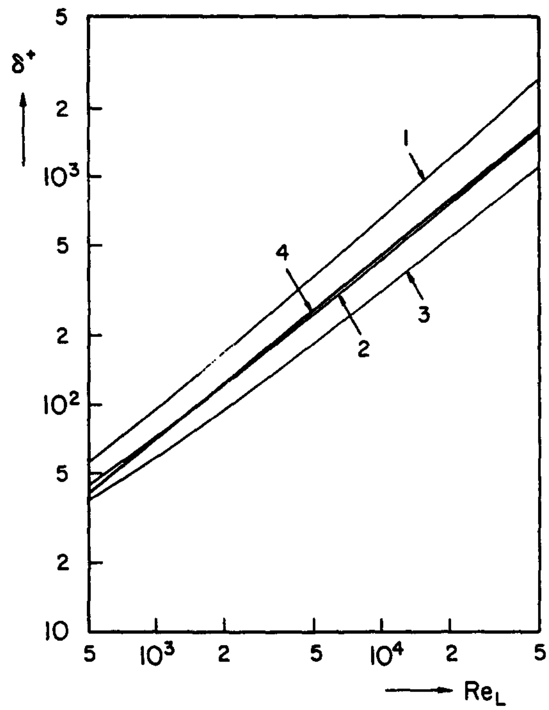


Figure 2. Dimensionless thickness of falling film. (1), this analysis ($m = 0.40$); (2), this analysis ($m = 0.18$); (3), this analysis ($m = 0.10$); (4), Brauer (1956).

0.18 is considerably smaller than that of 0.40 for single-phase flow. This, however, can be supported by the measurement of Koziol *et al.* (1980), who measured the maximum velocity in falling film flow.

2.2 *Annular flow*

The model for analysis is a simplified annular flow consisting of a liquid film and a gas core, as shown in figure 3. Entrainment is neglected and the liquid film is assumed to be smooth and to have uniform thickness in the circumferential direction. These assumptions are not always valid in actual cases. Therefore, this analysis may not be applied to annular flow with a considerable amount of entrainment or with large nonuniformity in film thickness. The shear stress distribution is

$$\tau = \left(1 - \frac{y}{R}\right)\tau_w, \qquad [6]$$

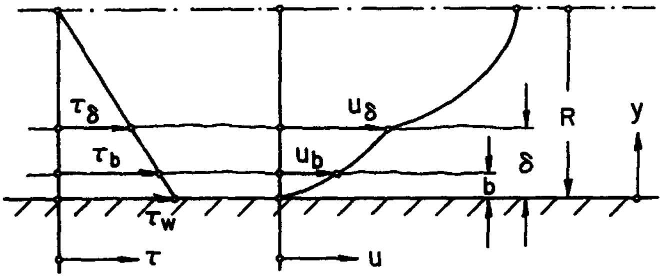


Figure 3. Model for annular flow.

where R is the pipe radius. The velocity profiles in the laminar sublayer and the turbulent layer are described as follows:

$$u = \frac{\tau_w}{\mu_L}\left(y - \frac{y^2}{2R}\right) \simeq \frac{\tau_w}{\mu_L} y, \text{ for } 0 \leq y \leq b, \tag{7}$$

$$u = u_b + \frac{\sqrt{\tau_w/\rho_L}}{m} \ln\left(\frac{y}{b}\right), \qquad \text{for } b \leq y \leq \delta. \tag{8}$$

Using the expression for the shear stress

$$\tau = \rho_G l^2 \left(\frac{du}{dy}\right)^2, \tag{9}$$

the velocity profile in the gas core yields to

$$u = u_\delta + \frac{\sqrt{\tau_w/\rho_G}}{m}\left(2\sqrt{1 - y/R} - 2\sqrt{1 - \delta/R} + \ln\frac{1 - \sqrt{1 - y/R}}{1 + \sqrt{1 - y/R}} - \ln\frac{1 - \sqrt{1 - \delta/R}}{1 + \sqrt{1 - \delta/R}}\right), \text{ for } \delta \leq y \leq R, \tag{10}$$

where ρ_G is the gas density. The mass flow rate for the liquid phase is

$$G(1 - x)\pi R^2 = 2\pi\rho_L \int_0^\delta u(R - y)\,dy \simeq 2\pi R\rho_L \int_0^\delta u\,dy, \tag{11}$$

and for the gas phase

$$Gx\pi R^2 = 2\pi\rho_G \int_\delta^R u(R - y)\,dy, \tag{12}$$

where G is the mass flux and x is the quality. Equations [7], [8] and [10]–[12] can be expressed in dimensionless forms as follows:

$$u^+ = y^+, \text{ for } 0 \leq y^+ \leq b^+, \tag{7a}$$

$$u^+ = u_b^+ + \frac{1}{m}\ln\left(\frac{y^+}{b^+}\right), \text{ for } b^+ \leq y^+ \leq \delta^+, \tag{8a}$$

$$u^+ = u_\delta^+ + \frac{\Gamma^{1/2}}{m}\left(2\sqrt{1 - 2y^+/\mathrm{Re}^+} - 2\sqrt{1 - 2\delta^+/\mathrm{Re}^+} + \ln\frac{1 - \sqrt{1 - 2y^+/\mathrm{Re}^+}}{1 + \sqrt{1 - 2y^+/\mathrm{Re}^+}} - \ln\frac{1 - \sqrt{1 - 2\delta^+/\mathrm{Re}^+}}{1 + \sqrt{1 - 2\delta^+/\mathrm{Re}^+}}\right), \text{ for } \delta^+ \leq y^+ \leq \frac{\mathrm{Re}^+}{2}, \tag{10a}$$

$$\frac{1}{4}\mathrm{Re}_L = \int_0^{\delta^+} u^+ dy^+, \text{ for liquid phase}, \tag{11a}$$

$$\frac{1}{4}\mathrm{Re}_L \frac{x}{1 - x}\Gamma = \int_{\delta^+}^{\mathrm{Re}^+/2}\left(1 - \frac{2y^+}{\mathrm{Re}^+}\right) u^+ dy^+, \text{ for gas phase}, \tag{12a}$$

where $\Gamma(=\rho_L/\rho_G)$ is the density ratio, $\mathrm{Re}^+(=\rho_L u^* d/\mu_L)$ the friction Reynolds number, $d(=2R)$ the pipe diameter and $\mathrm{Re}_L[=G(1 - x)d/\mu_L]$ the liquid Reynolds number.

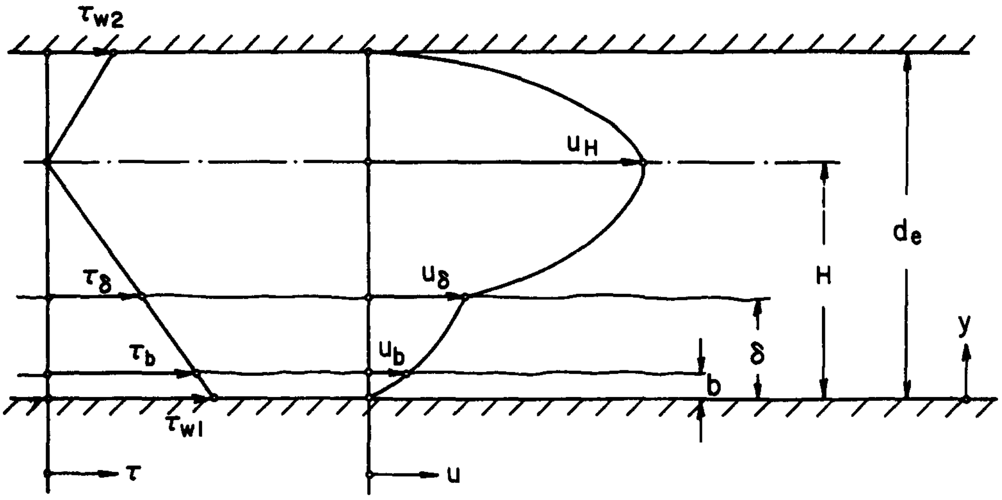


Figure 4. Model for stratified flow.

2.3 *Stratified flow*

For simplicity, a stratified flow is modeled as a flow between two parallel plates with distance d_e, as shown in figure 4. The relationship between d_e and the pipe diameter d will be determined afterward empirically.

Using the shear stress distribution

$$\tau = \frac{H-y}{H}\tau_{w1}, \text{ for } 0 \leq y \leq H, \qquad [13]$$

$$\tau = \frac{y-H}{d_e - H}\tau_{w2}, \text{ for } H \leq y \leq d_e, \qquad [14]$$

the velocity profile for the flow below the maximum velocity position ($y = H$) can be derived in the same manner as for the annular flow model:

$$u = \frac{\tau_{w1}}{\mu_L}\left(y - \frac{y^2}{2H}\right) \simeq \frac{\tau_{w1}}{\mu_L} y, \text{ for } 0 \leq y \leq b, \qquad [15]$$

$$u = u_b + \frac{\sqrt{\tau_{w1}/\rho_L}}{m}\left\{2\sqrt{1-y/H} - 2\sqrt{1-b/H} + \ln\frac{1-\sqrt{1-y/H}}{1+\sqrt{1-y/H}} - \ln\frac{1-\sqrt{1-b/H}}{1+\sqrt{1-b/H}}\right\}, \text{ for } b \leq y \leq \delta, \qquad [16]$$

$$u = u_\delta + \frac{\sqrt{\tau_{w1}/\rho_G}}{m}\left\{2\sqrt{1-y/H} - 2\sqrt{1-\delta/H} + \ln\frac{1-\sqrt{1-y/H}}{1+\sqrt{1-y/H}} - \ln\frac{1-\sqrt{1-\delta/H}}{1+\sqrt{1-\delta/H}}\right\}, \text{ for } \delta \leq y \leq H. \qquad [17]$$

For the velocity profile of the gas flow in the upper part of the maximum velocity position (neutral surface), one-seventh power law seems to be applicable;

$$\frac{u}{u_H} = \left(\frac{d_e - y}{d_e - H}\right)^{1/7}. \qquad [18]$$

Because this flow has solid and neutral boundaries, it can be probably treated as a single-phase flow in ducts. The mass flow rates for the liquid and gas phases are

$$G(1-x) = \rho_L \int_0^\delta u \, \mathrm{d}y, \tag{19}$$

$$Gx = \rho_G \int_\delta^{d_e} u \, \mathrm{d}y, \tag{20}$$

respectively, where G is the mass flow rate per unit width. The relationship between τ_{w1} and τ_{w2} is

$$\frac{\mathrm{d}P}{\mathrm{d}L} = \frac{\tau_{w1}}{H} = \frac{\tau_{w2}}{d_e - H}, \tag{21}$$

assuming that the pressure in the cross section is uniform. $\mathrm{d}P/\mathrm{d}L$ represents the frictional pressure drop per unit length. Equations [15]–[20] can be expressed in dimensionless forms as follows:

$$u^+ = 4\left(\frac{H^+}{\mathrm{Re}^+}\right) y^+, \text{ for } 0 \leq y^+ \leq b^+, \tag{15a}$$

$$u^+ = u_b^+ + \frac{2\sqrt{H^+/\mathrm{Re}^+}}{m}\left(2\sqrt{1 - y^+/H^+} - 2\sqrt{1 - b^+/H^+} + \ln\frac{1 - \sqrt{1 - y^+/H^+}}{1 + \sqrt{1 - y^+/H^+}} - \ln\frac{1 - \sqrt{1 - b^+/H^+}}{1 + \sqrt{1 - b^+/H^+}}\right), \text{ for } b^+ \leq y^+ \leq \delta^+, \tag{16a}$$

$$u^+ = u_\delta^+ + \frac{2\sqrt{(H^+/\mathrm{Re}^+)\Gamma}}{m}\left(2\sqrt{1 - y^+/H^+} - 2\sqrt{1 - \delta^+/H^+} + \ln\frac{1 - \sqrt{1 - y^+/H^+}}{1 + \sqrt{1 - y^+/H^+}} - \ln\frac{1 - \sqrt{1 - \delta^+/H^+}}{1 + \sqrt{1 - \delta^+/H^+}}\right), \text{ for } \delta^+ \leq y^+ \leq H^+, \tag{17a}$$

$$u^+ = u_H^+ \left(\frac{1 - y^+/\mathrm{Re}^+}{1 - H^+/\mathrm{Re}^+}\right)^{1/7}, \text{ for } H^+ \leq y^+ \leq \mathrm{Re}^+, \tag{18a}$$

$$\mathrm{Re}_L = \int_0^{\delta^+} u^+ \, \mathrm{d}y^+, \text{ for liquid phase}, \tag{19a}$$

$$\mathrm{Re}_L \frac{x}{1-x} \Gamma = \int_{\delta^+}^{\mathrm{Re}^+} u^+ \, \mathrm{d}y^+, \text{ for gas phase.} \tag{20a}$$

$H^+ (= \rho_L u^* H/\mu_L)$ represents the dimensionless maximum velocity position and $\mathrm{Re}_L[= G(1-x)/\mu_L]$ the liquid Reynolds number. If we assume that the value $b^+ = 7$ is valid also for the liquid flow in stratified flow, then b^+ in stratified flow is expressed as follows:

$$b^+ = \frac{7}{2\sqrt{H^+/\mathrm{Re}^+}}. \tag{22}$$

The position of maximum velocity is approximated to be located at the middle between the film surface and the upper plate for simplicity:

$$H^+ = \frac{\mathrm{Re}^+ + \delta^+}{2}. \tag{23}$$

The effect of this approximation will be included afterward in the relationship between d_e and d.

3. PRESSURE DROP CALCULATION BASED ON THIS ANALYSIS

The value of 0.18 for m has been shown to be valid for falling film flow. This value is assumed to be valid also for two-phase flow, i.e. for liquid film flow and gas flow, which have a wavy interface as a boundary. We can then calculate the frictional pressure drop of annular and stratified flow from the analysis in the previous sections.

3.1 *Annular flow*

Substituting [7a], [8a] and [10a] for the dimensionless velocity profile into [11a] and [12a], the two unknowns Re^+ and δ^+ can be solved for given values of Re_L, x and Γ using iteration procedure. Figure 5 shows the results. From Re^+, we can obtain the pressure drop from the definition of Re^+, u^* and the relationship between the wall shear stress and the pressure drop:

$$\frac{\mathrm{d}P}{\mathrm{d}L} = \frac{4}{d}\tau_w = \frac{4\,(\mathrm{Re}^+)^2\,\mu_L^2}{\rho_L\,d^3}. \qquad [24]$$

3.2 *Stratified flow*

In the same way as for annular flow, Re^+ and δ^+ can be calculated from [19a] and [20a] with [15a]–[18a] for given values of Re_L, x and Γ. Figure 6 shows the results.

In the case of stratified flow, flow in pipe was modeled as a flow between two parallel plates. Therefore, the relationship between the distance d_e and the pipe diameter d has to be found. Comparison was made with experimental data on refrigerants in a stratified flow region presented in part I, to seek a suitable equivalent value, with which the analytically obtained pressure drop agreed with experimental data. Figure 7 shows the results. It was

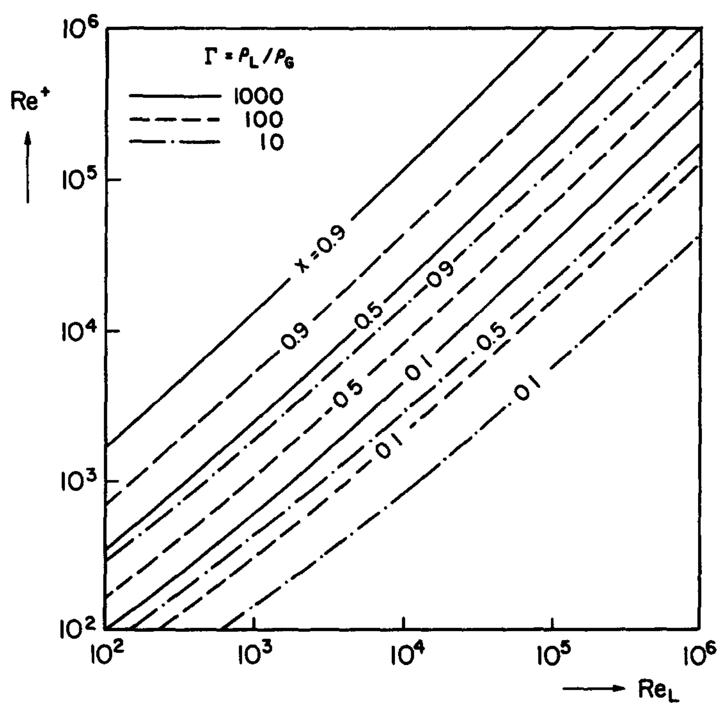


Figure 5. Friction Reynolds number after this analysis for annular flow.

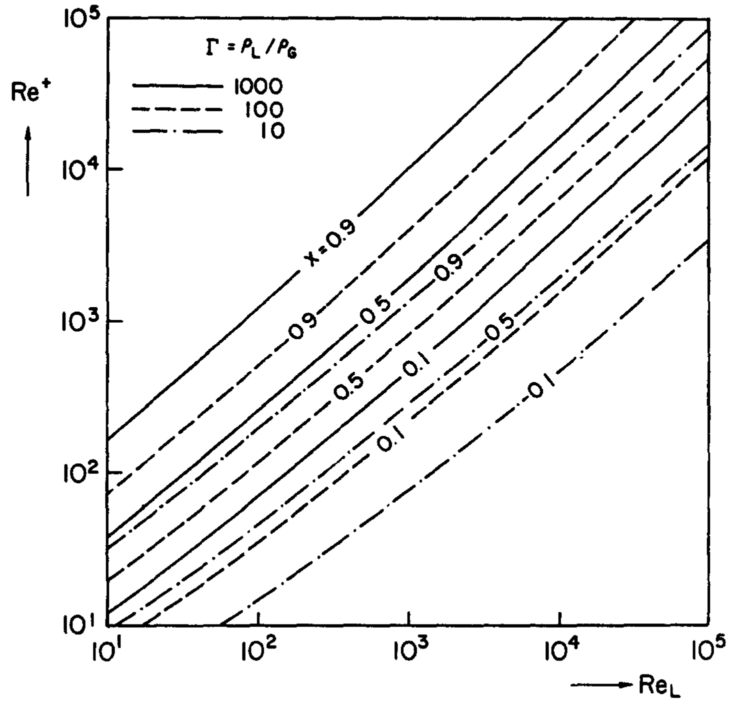
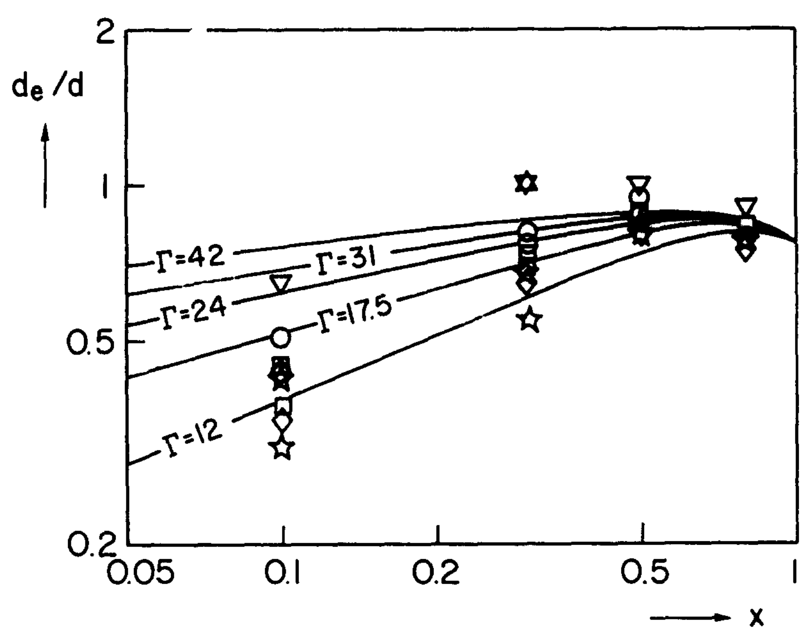


Figure 6. Friction Reynolds number after this analysis for stratified flow.



	T_s [°C]	Γ	
R12	20	42.2	△
	39	24.2	○
	50	17.7	◇
R22	20	31.3	▽
	39	17.2	□
	50	12.3	☆

Figure 7. Relationship between d_e and d. Data by Hashizume (1983a).

found that d_e/d is a function of x and Γ, and can be approximated as

$$\frac{d_e}{d} = [(x^{5.0/\Gamma})^{-2} + (1.22x^{-1.42})^{-2}]^{-0.5}. \quad [25]$$

The lines in figure 7 show the calculated values after [25]. The relationship includes such effects as curvature of the liquid surface, surface wave and the position of the maximum gas velocity, which were neglected or approximated in the analysis. Therefore, [25] is a pure empirical relationship and has no physical meaning.

3.3 *Flow pattern transition*

The variation of pressure drop versus mass flux under a constant quality shows characteristic slopes on a log-log plot. Each of these slopes corresponds to stratified, transient and annular flow. Bandel (1973) and Bandel & Schlünder (1973, 1974) found these characteristics for the first time, and proposed to calculate pressure drop in the transient region by interpolation between stratified and annular flow as follows:

$$\left(\frac{dP}{dL}\right)_{\text{transient}} = \left(\frac{dP}{dL}\right)_{SC} \times \left(\frac{G}{G_{SC}}\right)^{\log[(dP/dL)_{AC}/(dP/dL)_{SC}]/\log[G_{AC}/G_{SC}]}, \quad [26]$$

where the subscripts SC and AC denote the critical values for stratified and annular flow, respectively. This procedure is shown in figure 8. To do this, the critical mass flux G_{SC} and G_{AC} must be predicted. Bandel found criteria for them as functions of the product of Froude and Euler numbers empirically using experimental data on pressure drop of his own and from a data bank.

Another possibility to predict G_{SC} and G_{AC} is the use of some flow pattern maps. Hashizume (1983b) has obtained a flow pattern map, the modified Baker map, based on his own observation. The boundaries for the flow pattern transitions can be approximated by the following correlations:

Stratified to wavy: $Y = [(13.6X^{-0.1776})^{-1} + (972X^{-1.429})^{-1}]^{-1}$, [27]

Wavy to slug: $Y = 972X^{-1.429}$, [28]

Wavy to annular: $Y = 28.05X^{-0.1436}$. [29]

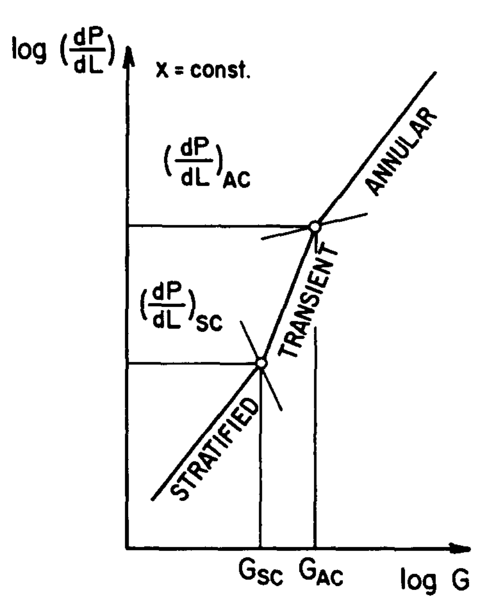


Figure 8. Linear interpolation for transient flow region according to Bandel (1973) and Bandel & Schlünder (1973, 1974).

Here X, Y, λ and ψ' are

$$X = \frac{1-x}{x}\lambda\psi' \quad (-), \tag{30}$$

$$Y = \frac{Gx}{\lambda} \quad \left(\frac{\mathrm{kg}}{m^2\mathrm{s}}\right), \tag{31}$$

$$\lambda = \left[\left(\frac{\rho_G}{\rho_{AA}}\right)\left(\frac{\rho_L}{\rho_{WA}}\right)\right]^{1/2}, \tag{32}$$

$$\psi' = \left(\frac{\sigma_{WA}}{\sigma_L}\right)^{1/4}\left[\left(\frac{\mu_L}{\mu_{WA}}\right)\left(\frac{\rho_{WA}}{\rho_L}\right)^2\right]^{1/3}, \tag{33}$$

where σ represents the surface tension and the subscripts AA and WA denote the air and water at ambient conditions, respectively.

4. COMPARISONS OF THIS ANALYSIS WITH EXPERIMENTAL DATA OF REFRIGERANT TWO-PHASE FLOW IN HORIZONTAL PIPES

4.1 *Pressure drop*

Figures 9–12 show the comparisons with the experimental data by Hashizume (1973a), Chawla (1967) and Bandel (1973). Pressure drop in the wavy flow region is interpolated between stratified and annular flow, and that in the slug flow region is calculated using the annular flow model. Though flow pattern transitions shown in these figures after [27]–[29] do not always agree with the observations, we can see good agreements in pressure drop between this analysis and the experimental data.

This analysis is compared with other calculation methods of frictional pressure drop (Bandel 1973; VDI-Wärmeatlas 1974; Friedel 1979). Comparisons are made in the form of relative errors defined as

$$\frac{(\mathrm{d}P/\mathrm{d}L)_{\mathrm{calculated}} - (\mathrm{d}P/\mathrm{d}L)_{\mathrm{measured}}}{(\mathrm{d}P/\mathrm{d}L)_{\mathrm{calculated}}} \times 100, \tag{34}$$

and the percentage-wise fraction of data within $\pm 30\%$ relative errors is given in table 1. The table shows that this analysis can predict pressure drop more accurately than the other calculation methods, at least in this experimental range. The VDI-Wärmeatlas method predicts Chawla's data most accurately, but agreements with other data are not satisfactory. Bandel's method gives good prediction except of Chawla's data of 25-mm diameter, which was already recognized by Bandel himself. Friedel's method was developed for steam/water ($P = 1$–212 bar, $G = 15$–8250 kg/m^{2}s, $d = 3$–152 mm and $x < 0.7$) and R12($P = 0.6$–33 bar, $G = 100$–5000 kg/m^{2}s, $d = 10$–23 mm and $x < 0.7$). Although most data on R12 lie in these applicable ranges, agreement between this method and experimental data is unsatisfactory.

4.2 *Void fraction*

The simultaneous equations, i.e. [11a] and [12a] for annular flow and [19a] and [20a] for stratified flow, give two answers. One answer is Re^+, which was verified by comparisons with systematically performed experimental data. Another answer is δ^+. From the liquid film thickness δ, void fraction α can be calculated, if entrainment is neglected. That is

$$\alpha = \frac{\pi(R-\delta)^2}{\pi R^2} \simeq 1 - 2\frac{\delta}{R} = 1 - \frac{4\delta^+}{\mathrm{Re}^+} \tag{35}$$

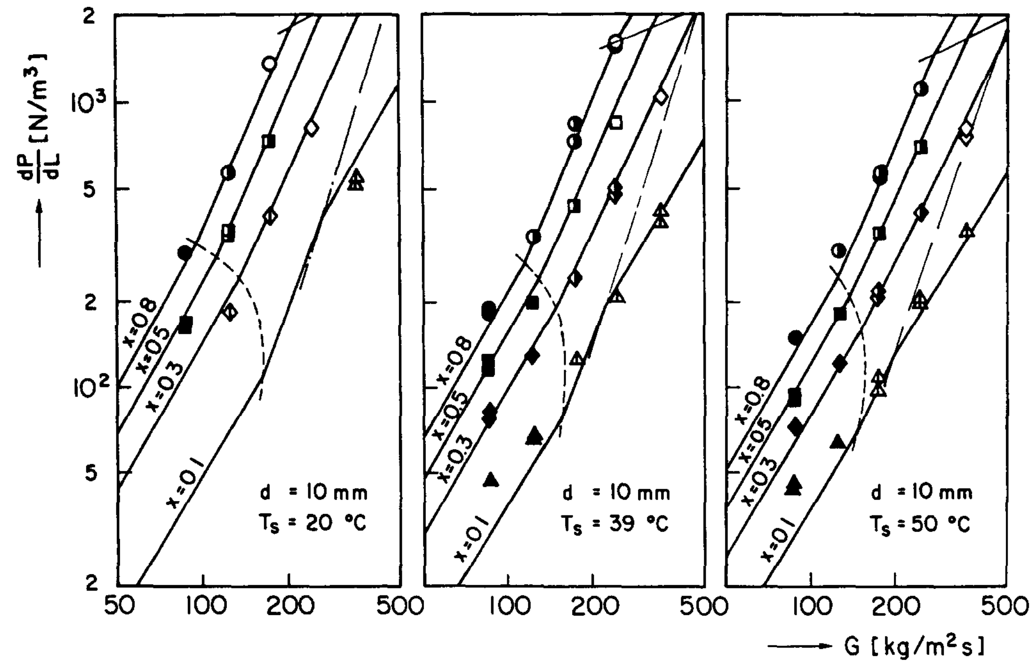


Figure 9. Comparison of this analysis with R12 data by Hashizume (1983a). Symbols and lines for figures 9–12 shown below.

QUALITY x				OBSERVED FLOW PATTERN
0.1	0.3	0.5	0.8	
▲	◆	■	●	STRATIFIED FLOW
△	◇	□	○	WAVY FLOW
△	◇	□	○	SLUG FLOW
△	◇	□	○	ANNULAR FLOW

FLOW PATTERN TRANSITION ACCORDING TO [27]–[29]

- – – – – STRATIFIED TO WAVY
- —·— – WAVY TO SLUG
- ——— WAVY TO ANNULAR

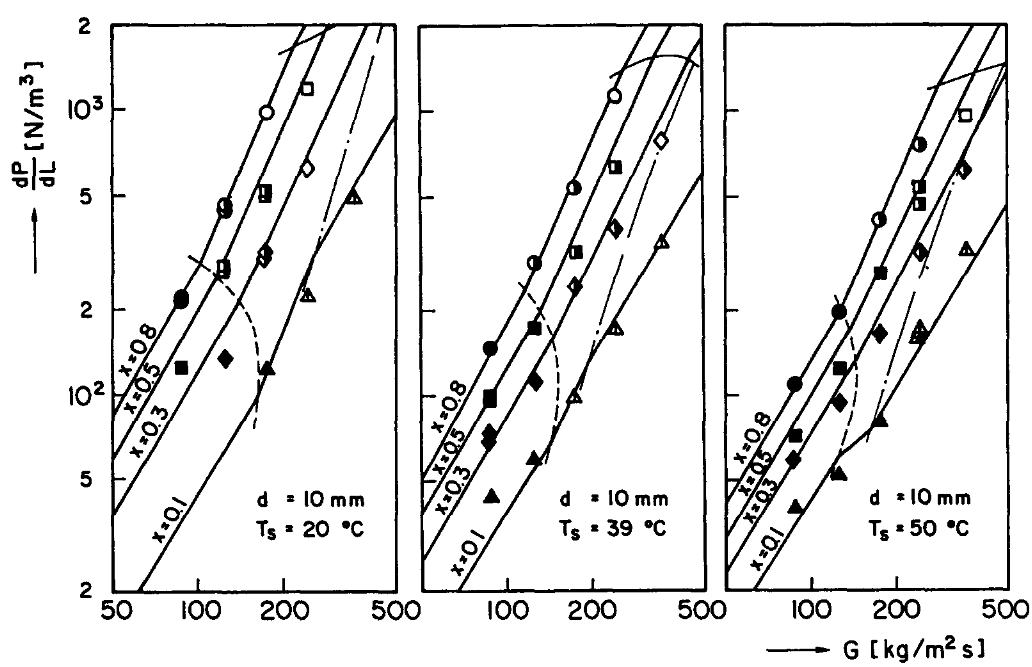


Figure 10. Comparison of this analysis with R22 data by Hashizume (1983a).

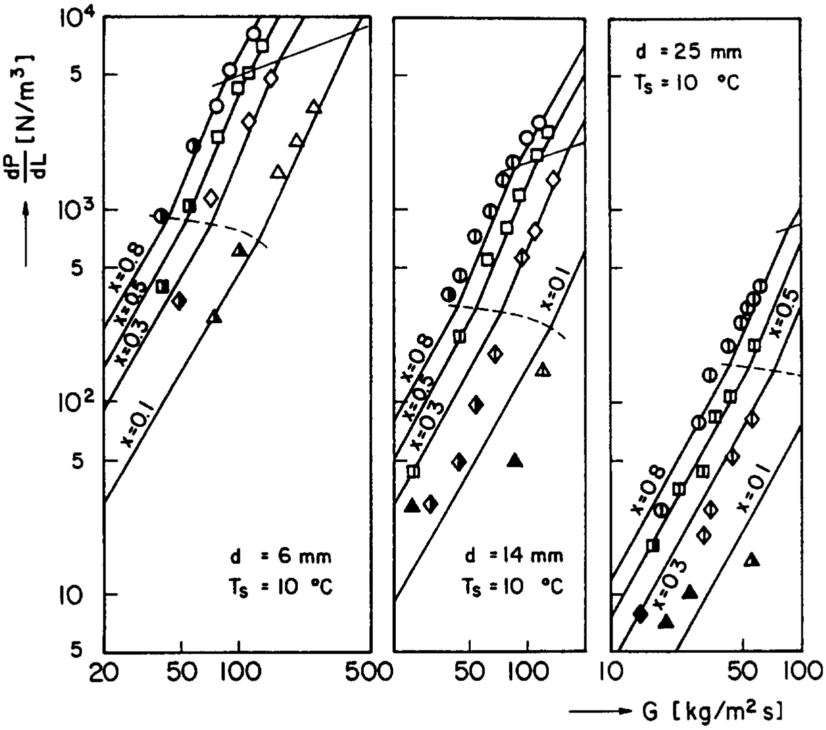


Figure 11. Comparison of this analysis with R11 data by Chawla (1967).

for annular flow and

$$\alpha = \frac{d_e - \delta}{d_e} = 1 - \frac{\delta}{d_e} = 1 - \frac{\delta^+}{\mathrm{Re}^+} \tag{36}$$

for stratified flow.

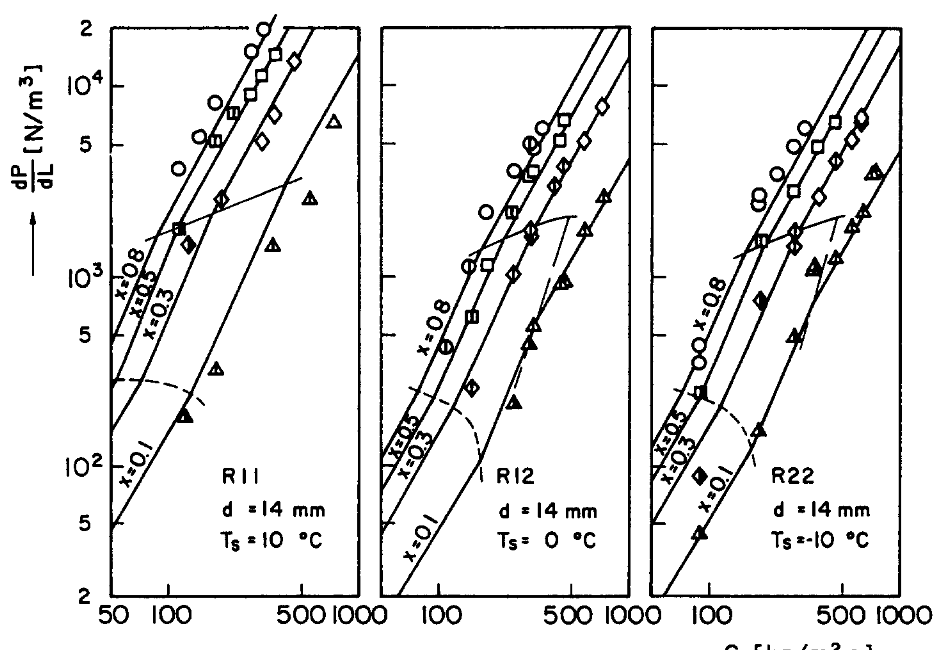


Figure 12. Comparison of this analysis with refrigerant data by Bandel (1973).

Table 1. Comparison of frictional pressure drop calculation methods with experimental data

Data \ Calculation Method				This Analysis	VDI-Wärmeatlas (1974)	Bandel (1973) Bandel & Schlünder (1973,1974)	Friedel (1979)
Author	Refrigerant	d [mm]	T_s [°C]				
Hashizume (1983a)	R12	10	20	100	30	80	10
			39	94	19	78	13
			50	94	27	82	15
	R22		20	100	12	92	15
			39	96	13	96	8
			50	100	37	100	11
Chawla (1967)	R11	6	10	92	96	81	0
		14		66	77	63	23
		25		70	82	33	0
Bandel (1973)	R11	14	10	68	46	77	50
	R12		0	72	39	50	64
	R22		-10	66	28	53	63

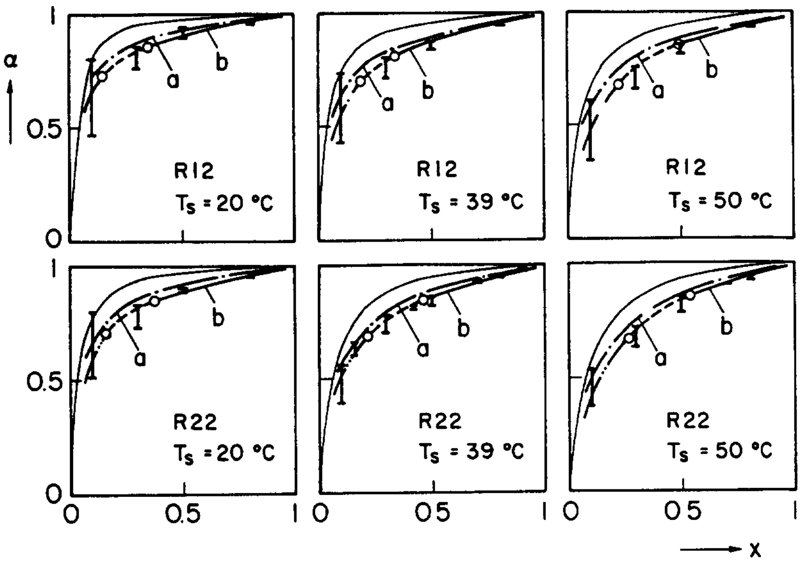


Figure 13. Void fraction compared with data by Hashizume (1983a).

O	Flow Pattern Transition According to [27]-[29]
—·—	Calculated by Stratified Flow Model
- - - - -	Interpolated (Wavy Flow)
—··—	Calculated by Annular Flow Model (Slug Flow)
———	Calculated by Annular Flow Model
———	Calculated by Homogeneous Flow Model
I	Range of Experimental Data

a Calculated for G = 88.4 kg/m²s (= 25 kg/h)

b Calculated for G =353.7 kg/m²s (=100 kg/h)

To evaluate this analysis from another point of view, void fraction calculated by [35] or [36] is compared with experimental data presented in part I. Figure 13 shows the results. In the annular flow region, good agreement is seen. In the stratified flow region, however, agreement is not as good as in the annular flow region, because the relationship between d_e and d [25] was determined to fit the pressure drop. In spite of this, we can see better agreements at least than the well-known homogeneous flow model. Lines for the wavy flow region are smooth interpolation between stratified and annular flow model. The flow pattern transitions were determined here also by [27]–[29].

5. CONCLUSION

Two-phase flow in horizontal pipe was analyzed using simplified models for annular and stratified flow. The velocity profile for the turbulent flow was described with the Prandtl mixing length. From the analysis of falling film flow, the mixing length constant for the liquid film flow was found to be 0.18 instead of 0.40 for single-phase flow. With the extrapolation of this value to gas flow of two-phase flow, the frictional pressure drop of annular and stratified flow could be calculated as an answer of simultaneous equations for liquid and gas flow rates. The intermediate region between them, i.e. wavy flow, was interpolated with flow pattern transition by the modified Baker map. Comparisons of this analysis with existing experimental data on refrigerants showed good agreements.

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NOMENCLATURE

b	laminar sublayer thickness
b^+	dimensionless thickness of laminar sublayer $= \rho_L u^* b/\mu_L$
d	pipe diameter
d_e	distance between upper and lower plates
$\mathrm{d}P/\mathrm{d}L$	pressure drop per unit length
g	gravity acceleration
G	mass flux
H	maximum velocity position in stratified flow model
H^+	dimensionless maximum velocity position in stratified flow model $= \rho_L u^* H/\mu_L$
l	mixing length
L	length
m	mixing length constant
P	pressure
R	pipe radius $= d/2$
Re^+	friction Reynolds number (d_e is used for stratified flow model) $= \rho_L u^* d/\mu_L$
Re_L	liquid Reynolds number $= G(1-x)\,d/\mu_L$
	for falling film flow $= G/\mu_L$
	for stratified flow model $G(1-x)/\mu_L$
T_s	saturation temperature
u	velocity
u^+	dimensionless velocity $= u/u^*$
u^*	friction velocity $= \sqrt{\tau_w/\rho_L}$
	for falling film flow $= \sqrt{g\delta}$
	for stratified flow model $= \sqrt{d_e/4\rho_L\,\tau_{wl}/H}$
x	quality
y	distance from the wall
y^+	dimensionless distance $= \rho_L u^* y/\mu_L$
α	void fraction
Γ	density ratio $= \rho_L/\rho_G$
δ	liquid film thickness
δ^+	dimensionless liquid film thickness $= \rho_L u^*\delta/\mu_L$
μ	viscosity
ρ	density
σ	surface tension
τ	shear stress

Subscripts

AA	air at ambient conditions
AC	critical value for annular flow
b	laminar sublayer
G	gas phase
H	maximum velocity position in stratified flow model
L	liquid phase
SC	critical value for stratified flow
w	wall
WA	water at ambient conditions
δ	liquid film surface

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